NATURAL ENTROPY PRODUCTION IN AN INFLATIONARY MODEL FOR A POLARIZED VACUUM

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Abstract

Though entropy production is forbidden in standard FRW Cosmology, Berman and Som presented a simple inflationary model where entropy production by bulk viscosity, during standard inflation without ad hoc pressure terms can be accommodated with Robertson-Walker's metric, so the requirement that the early Universe be anisotropic is not essential in order to have entropy growth during inflationary phase, as we show. Entropy also grows due to shear viscosity, for the anisotropic case. The intrinsically inflationary metric that we propose can be thought of as defining a polarized vacuum, and leads directly to the desired effects without the need of introducing extra pressure terms.

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I. Introduction

[Note to the reader: this paper was originally prepared around the year 1990. It was submitted in an earlier version, the referee asked for clarifications, but for some reason, we made the clarifying in text, but did not re-submit. As it seems to us that the contents is still adequate, we publish it, at this time.]

Standard FRW Cosmology models do not provide a mechanism for entropy production; unless one modifies the pressure by including an ad-hoc viscosity term, like in Berman and Paim[6]. However, Berman and Som[1] presented a special anisotropic model of the Bianchi type I that undergoes exponential expansion and leads to entropy production directly, without further assumptions. The geometry of the model during inflation is described by metric:

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \quad , \tag{1.1}$$

with

$$A = A_0 e^{D_1 t} \quad , \tag{1.2a}$$

$$B = B_0 e^{D_2 t} \quad , \tag{1.2b}$$

$$C = C_0 e^{D_3 t} \quad , \tag{1.2c}$$

where A_0, B_0, C_0 and D_i 's are constants $(i = 1, 2, 3)$. This metric can be thought of as representing a polarized vacuum.

The content of the model is the constant energy density ρ , isotropic constant pressure p and the trace-free constant anisotropic pressure Π_i^k given by, in units $\frac{8\pi G}{c^4} = 1$, as:

$$\rho = D_1 D_2 + D_2 D_3 + D_3 D_1 \quad , \tag{1.3}$$

$$p = -\frac{1}{3}\left[\rho + 2\left(D_1^2 + D_2^2 + D_3^3\right)\right] \quad , \tag{1.4}$$

$$\Pi_i^i = -3D\sigma_i^i \qquad \text{(no summation)} \tag{1.5}$$

where σ_i^i is the shearing tensor and $D = \frac{1}{3}\left(D_1 + D_2 + D_3\right)$. We see from (1.5) that the coefficient of viscosity due to shear is a constant.

The negative isotropic pressure is the usual phenomenon associated with the exponential expansion,

$$H = \frac{1}{3}\left(D_1 + D_2 + D_3\right) = D \quad .$$

The conservation of thermal energy is then given by[2]:

$$T\dot{S} = D\sigma_k^i \sigma_i^k \quad ,$$

where S is the entropy density and T is the temperature. It would be very attractive if we could explain the huge entropy per baryon, denoted by the microwave background, by physical processes acting in the early, or very early, Universe, as was remarked by Weinberg[3]. The present work is a step towards that goal, and it will be shown that the requirement that the Universe undergoes anisotropic inflation is not essential to entropy production, because it does happen in the isotropic case, for isotropic inflation.

II. The entropy production due to shear viscosity

To obtain the entropy production, we define the entropy current according to the second order dissipative relativistic fluid theories of Israel[4], appropriate to this model,

$$S^\mu = \frac{U^\mu}{T}\left[TS - \frac{1}{2}\beta_2 \Pi^{\alpha\beta}\Pi_{\alpha\beta}\right] \quad , \tag{2.1}$$

where β_2 is the phenomenological coefficient, S is the equilibrium entropy density and U^μ is the 4-velocity,

$$U^\mu U_\mu = 1 \quad . \tag{2.2}$$

Taking the divergence of (2.1), one shall obtain, supposing that the time derivative of β_2 is of first order,

$$S^{\mu};_{\mu} = -\frac{\Pi^{\beta}_{\alpha}}{T}\left(U^{\alpha}_{;\beta} + \beta_2 \dot{\Pi}^{\alpha}_{\beta}\right) \qquad , \tag{2.3}$$

where $\dot{\Pi}_{\alpha\beta} = \Pi_{\alpha\beta;\lambda} U^{\lambda}$, the semicolon denoting covariant derivative.

From (2.1), we find (2.3) by means of the following intermediate steps:

$$S^{\mu} = \frac{U^{\mu}}{T}\left[TS - \frac{1}{2}\beta_2 \Pi^{\alpha\beta}\Pi_{\alpha\beta}\right] = \frac{1}{T}\left[pU^{\mu} - U_{\lambda}T^{\lambda\mu}\right] - \frac{\beta_2}{2T}\Pi^{\alpha\beta}\Pi_{\alpha\beta}U^{\mu} \qquad .$$

We now find:

$$S^{\mu}_{;\mu} = -\frac{1}{T}U_{\lambda}T^{\lambda\mu}_{\ \ ;\mu} + \left(\frac{p}{T}U^{\mu}\right)_{;\mu} - \left(\frac{U_{\lambda}}{T}\right)_{;\mu} T^{\lambda\mu} - \left[\frac{1}{2T}\beta_2 \Pi^{\alpha\beta}\Pi_{\alpha\beta}U^{\mu}\right]_{;\mu} =$$

$$= -T^{-1}U_{\lambda}T^{\lambda\mu}_{\ \ ;\mu} - T^{-1}\Pi^{\lambda\mu}U_{\lambda;\mu} - \left[\frac{1}{2T}\beta_2 \Pi^{\alpha\beta}\Pi_{\alpha\beta}U^{\mu}\right]_{;\mu} \qquad . \tag{2.3b}$$

In the case of interest to us, the energy momentum tensor is covariant divergence-less:

$$T^{\lambda\mu}_{\ \ ;\mu} = 0 \qquad .$$

In calculating the last term in (2.3b), we can treat $U^{\mu}_{;\mu}$ and the time derivatives of $\frac{\beta_2}{T}$, as the first order quantities, since they vanish for equilibrium[7]. Hence, one obtains:

$$TS^{\mu}_{;\mu} = -\Pi^{\alpha\beta}\left(U_{\alpha;\beta} + \beta_2 \dot{\Pi}_{\alpha\beta}\right) \qquad .$$

Equation (2.3) gives the rate of entropy production per unity volume. The second law of thermodynamics $(S^{\mu};_{\mu} \geqslant 0)$ will be satisfied identically if:

$$\Pi^{\alpha}_{\beta} = -2n\left[U^{\alpha};_{\beta} + \beta_2 \dot{\Pi}^{\alpha}_{\beta}\right] \qquad , \tag{2.4}$$

where the positive coefficient of proportionality n is the coefficient of the shear viscosity.

From (1.5) we have $\dot{\Pi}^{\alpha}_{\beta} = 0$, and $2n = 3D =$ constant. Thus in Bianchi I type model the entropy grows in a constant rate. The coefficient of shear viscosity is $n = \frac{3}{2}H$.

During the period of inflation, the physical entropy is given by:

$$S^0 = S - \frac{\tau D^2}{T} \sigma_{\alpha\beta}\ \sigma^{\alpha\beta} \quad , \tag{2.5}$$

where S is the equilibrium entropy density, and,

$$\tau = \frac{9}{2}\beta_2 \quad .$$

III. Entropy production in the isotropic case

Let us consider the case of a small anisotropy such as:

$$D_1 = D_0 + \alpha \quad ,$$

$$D_2 = D_0 + \beta \quad ,$$

$$D_3 = D_0 + \gamma \quad ,$$

where α, β and γ are small quantities.

From (1.3) and (1.4) one obtains, neglecting the second order terms,

$$\rho \cong 3D_0^2 + 6D_0(\alpha + \beta + \gamma) \quad , \tag{3.1}$$

$$p = -\frac{1}{3}\left[9D_0^2 + 10D_0(\alpha + \beta + \gamma)\right] \quad . \tag{3.2}$$

From (3.1) and (3.2) one finds that:

$$p = -3D_0^2 - \frac{10}{3}D_0(\alpha + \beta + \gamma) \quad , \tag{3.3}$$

up to the first order.

In the isotropic exponential expansion one has:

$$p = -3D_0^2 = -\rho \quad , \tag{3.4}$$

which is the usual vacuum state condition for the inflationary model.

If we impose the condition $p = -\rho$, then relation (1.4) yields:

$$D_1^2 + D_2^2 + D_3^2 = D_1D_2 + D_2D_3 + D_3D_1 \quad , \tag{3.5}$$

leading to $D_1 = D_2 = D_3$. However, for a vacuum state cosmology in a Bianchi-I space-time, one needs, apart from a shear viscosity, a bulk viscosity given by:

$$\Pi = -\frac{2}{3}\left[D_1 D_2 + D_2 D_3 + D_3 D_1 + D_1 + D_2 + D_3\right] \quad . \qquad (3.6)$$

This leads to the modification of (2.1). The entropy current takes the form:

$$S^\mu \approx SU^\mu - \frac{1}{2}\left[\beta_0 \Pi^2 + \beta_2 \Pi^{\alpha\beta}\Pi_{\alpha\beta}\right]\frac{U^\mu}{T} \quad , \qquad (3.7)$$

where β_0 is a phenomenological coefficient. The inclusion of the bulk viscosity further augments the rate of production of entropy in the general case.

IV. Conclusions

The requirement that the early Universe be anisotropic is not essential in order to have entropy growth during inflation. We showed that entropy grows, due to bulk and shear viscosity, in the anisotropic case. An estimate of the maximum rate at which the entropy of the primordial plasma increases in the presence of a homogeneous shear, and the minimum entropy increase, were calculated by Vischniac[5]; who also discussed the physical mechanism underlying our proposed shear viscosity, which can be achieved through a massive scalar field interacting with a massive gauge field. The interest in our model resides in the natural way of entropy production, without adding ad-hoc viscosity terms to the pressure; all one has to do is define the vacuum polarization metric (1.1) and apply Einstein's equations.

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